Sampling Distribution of a Sample Proportion

Lecture 25
Sections 8.1 - 8.2

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Outline

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- We will simulate this situation.

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- Repeat this many times until we can see the shape of the distribution.

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Experimental Results

Count	Samp. Prop.	Frequency
6	0.24	3
7	0.28	6
8	0.32	15
9	0.36	31
10	0.40	52
11	0.44	68
12	0.48	85
13	0.52	73
14	0.56	62
15	0.60	44
16	0.64	33
17	0.68	19
18	0.72	5
19	0.76	3
20	0.80	1

I used `randBin(25,.5,25)` twenty times (seed = 161).

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- In our experiment, the value of $\hat{p}$ varied from sample to sample.
- $\hat{p}$ was as low as 0.24 and as high as 0.80.

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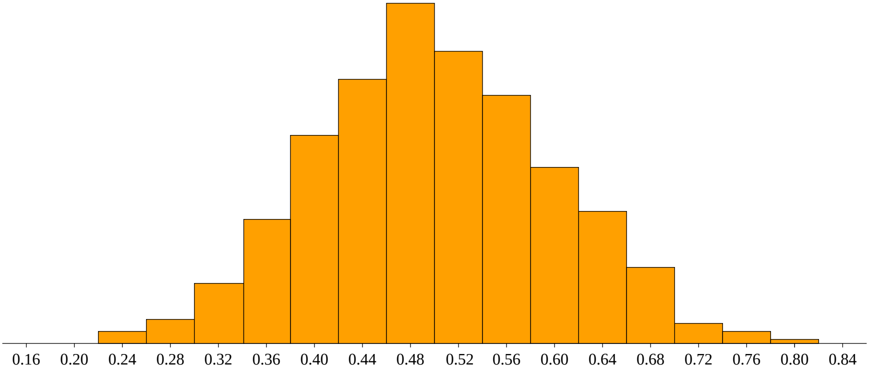
- Our experimental results again were very close!

Experimental Results

- We also notice that the distribution of values was nearly normal.

Distributions



Our results

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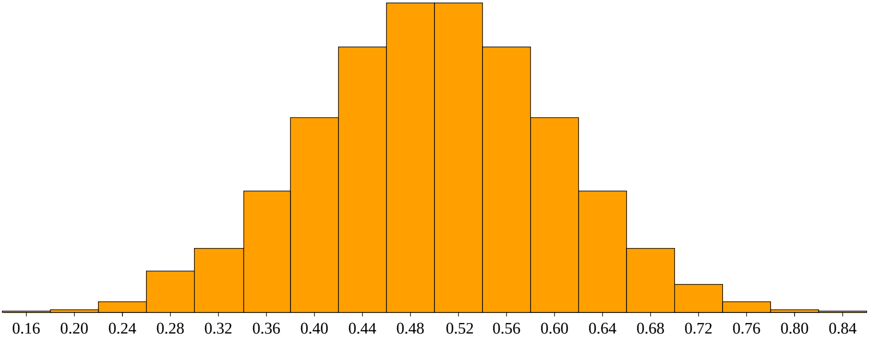
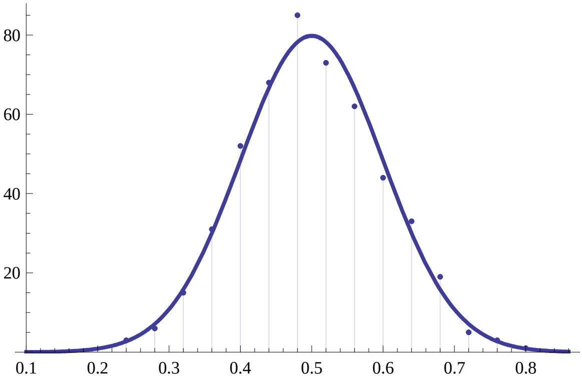


Theoretical results

Distributions



A normal curve

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$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

- It also says that for *large* samples, $\hat{p}$ has a normal distribution.
- This is called the Central Limit Theorem for Proportions.

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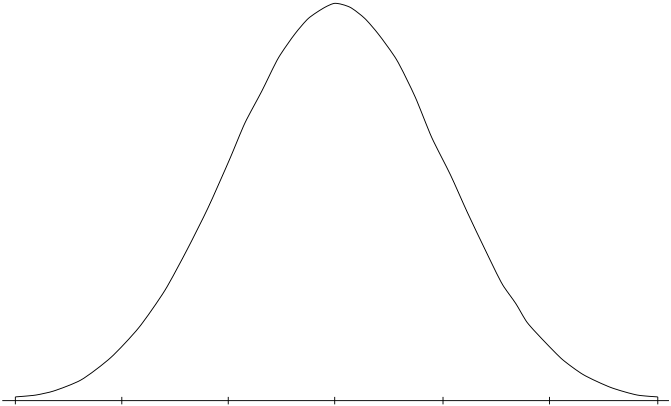

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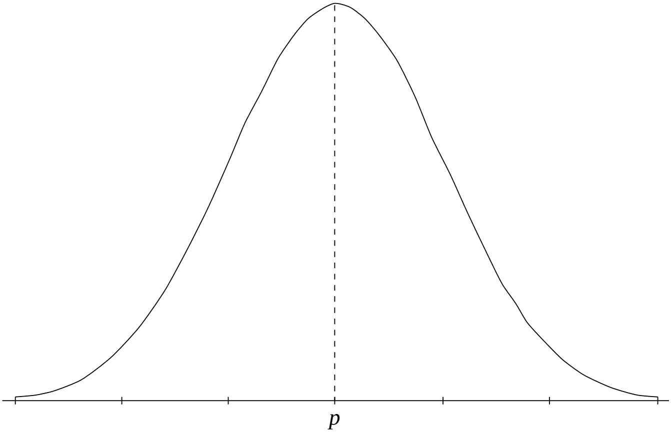

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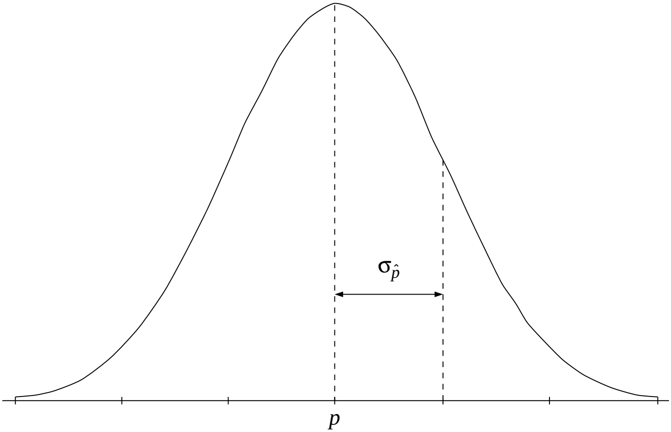

Example (Sampling Distribution of $\hat{p}$)

- Suppose the president's approval rating is 50%.
- If we sample 100 people, how likely is it that our sample proportion will be between 45% and 55%?

Example (Sampling Distribution of $\hat{p}$)

- From the Central Limit Theorem, we know that $\hat{p}$ has a normal distribution with mean 0.50 and standard deviation

$$\sqrt{\frac{(0.50)(0.50)}{100}} = 0.05.$$

- So the probability that $\hat{p}$ will be between 0.45 and 0.55 is

$$\mathit{normalcdf}(.45, .55, .50, .05) = 0.6827.$$

Example (Sampling Distribution of $\hat{p}$)

- Suppose the president's approval rating is 45%.
- Suppose we sample only 10 people, how likely is it that our sample proportion will be between 40% and 60%?
- This is harder.

Sampling Distributions of $\hat{p}$

Example (Sampling Distribution of $\hat{p}$)

- Suppose the president's approval rating is 45%.
- Suppose we sample only 10 people, how likely is it that our sample proportion will be between 40% and 60%?
- This is harder.
- Why?

Outline

Assignment

Homework

1. Given the population in which 25% strongly approve of the president's performance,
   (a) Find the sampling distribution of $\hat{p}$ for samples of size 3.
   (b) Use the Central Limit Theorem to find the mean and standard deviation of that distribution.
2. Repeat the previous exercise for a population in which 10% strongly approve of the president's performance.
3. Use the Central Limit Theorem to find the distribution of $\hat{p}$ when the sample size is $n = 100$.